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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. ***Use of statistical tables is permitted.***

PART – A

1 a. Using Taylor series method, solve the problem $\frac{dy}{dx} = x^2y - 1,\ y(0) = 1$ at the point x = 0.2. Consider upto 4^{th} degree terms. **(06 Marks)**

b. Using R.K. method of order 4, solve $\frac{dy}{dx} = 3x + \frac{y}{2},\ y(0) = 1$ at the points x = 0.1 and x = 0.2 by taking step length h = 0.1. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2,\ y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795,\ y(0.6) = 0.1762$. Compute y at x = 0.8 by Adams-Bashforth predictor-corrector method. Use the corrector formula twice. **(07 Marks)**

2 a. Evaluate y and z at x = 0.1 from the Picards second approximation to the solution of the following system of equations given by y = 1 and z = 0.5 at x = 0 initially.

$$\frac{dy}{dx} = z, \quad \frac{dz}{dx} = x^3(y + z)$$ **(06 Marks)**

b. Given $y'' - xy' - y = 0$ with the initial conditions y(0) = 1, y′(0) = 0. Compute y(0.2) and y′(0.2) by taking h = 0.2 and using fourth order Runge-Kutta method. **(07 Marks)**

c. Applying Milne's method compute y(0.8). Given that y satisfies the equation $y'' = 2yy'$ and y and y′ are governed by the following values. y(0) = 0, y(0.2) = 0.2027, y(0.4) = 0.4228, y(0.6) = 0.6841, y′(0) = 1, y′(0.2) = 1.041, y′(0.4) = 1.179, y′(0.6) = 1.468. (Apply corrector only once). **(07 Marks)**

3 a. Derive Cauchy Riemann equations in Cartesian form. **(06 Marks)**

b. Find an analytic function f(z) = u + iv. Given $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$. **(07 Marks)**

c. If f(z) is a regular function of z, show that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4\ |f'(z)|^2$ **(07 Marks)**

4 a. Find the bilinear transformation that maps the points z = -1, i, -1 onto the points w = 1, i, -1 respectively. **(06 Marks)**

b. Find the region in the w-plane bounded by the lines x = 1, y = 1, x + y = 1 under the transformation $w = z^2$. Indicate the region with sketches. **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)} dz$ where c is the circle |z| = 3. **(07 Marks)**

PART – B

5 a. Solve the Laplaces equation in cylindrical polar coordinate system leading to Bessel differential equation. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Express the polynomial, $2x^3 - x^2 - 3x + 2$ interms of Legendre polynomials. **(07 Marks)**

6 a. State and prove addition theorem of probability. **(06 Marks)**

b. Three students A, B, C write an entrance examination. Their chances of passing are ½, ⅓, ¼ respectively. Find the probability that,
i) Atleast one of them passes.
ii) All of them passes.
iii) Atleast two of them passes. **(07 Marks)**

c. Three machines A, B, C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective outputs of these three machines are respectively 2%, 3% and 4%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C. **(07 Marks)**

7 a. The pdf of a random variable x is given by the following table:

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

Find: i) The value of k ii) $P(x > 1)$ iii) $P(-1 < x \leq 2)$
iv) Mean of x v) Standard deviation of x. **(06 Marks)**

b. In a certain factory turning out razar blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing, i) One defective, ii) Two defective, in a consignment of 10000 packets. **(07 Marks)**

c. In a normal distribution 31% of items are under 45 and 8% of items are over 64. Find the mean and standard deviation of the distribution. **(07 Marks)**

8 a. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39350 kilometers with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40000 kilometers? (Use 0.05 level of significance) Establish 99% confidence limits within which the mean life of tyres expected to lie. (Given that $Z_{0.05} = 1.96$, $Z_{0.01} = 2.58$) **(06 Marks)**

b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (Given that $t_{0.05} = 2.262$ for 9 d.f.) **(07 Marks)**

c. Fit a Poisson distribution to the following data and test the goodness of fit at 5% level of significance. Given that $\psi^2_{0.05} = 7.815$ for 4 degrees of freedom.

x	0	1	2	3	4
Frequency	122	60	15	2	1

(07 Marks)

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